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APPLICATION OF WAVELET-COSINE TRANSFORM TO IMAGE COMPRESSION

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ABSTRACT

A two-dimensional discrete transform which is a composition of the wavelet and the cosine transform is presented and applied to image compression. The usefulness of the 2-D wavelet and wavelet-cosine transform for the lossy compression is compared. Independently of which transform was used, the nonuniform quantization distinctly increased the efficiency of lossy compression.

1 INTRODUCTION

The standard compressions [3] such as JPEG used the cosine transform (CT). JPEG 2000 [1][2] implements a new method of compressing images based on the wavelet transform (WT). The properties of this transform make it very useful in compression standards. Its frequency character enable us to exploit the knowledge connected with the frequency properties of human hearing and sight. Moreover the local character of WT does not introduce the block effects and makes it possible to conduct the compression in real time systems.

In this paper the application of a new transform, which combines both these transforms, is proposed. For a given image, the wavelet transform is applied first and next the cosine transform is computed. In this way we obtain the wavelet-cosine transform (WCT) values. Some properties and applications of the 1-D wavelet-Fourier and wavelet-cosine transforms are presented in papers [4] - [7].

To compress an image, its WCT spectrum is quantizied in smaller bits than the original image. It was observed that the compression efficiency strongly depends on the way of quantization. The last part of paper presents the results of experiments with uniform and nonuniform quantization obtained for Lena image.

2 DISCRETE 2-D WAVELET-COSINE TRANSFORM

Let us assume that images to be transformed are described by the series

$$s_{m+1}(x,y) = \sum_{n_1}\sum_{n_2} c_{m+1,n_1,n_2}\phi_{m+1,n_1,n_2}(x,y) \quad (1)$$

where the scale functions $\phi_{m+1,n_1,n_2}(x,y)$ generate the $m+1$ resolution level image. A given image s_{m+1} can be split into four images

$$\begin{aligned} s_{m+1}(x,y) = & \textstyle\sum_{n_1}\sum_{n_2} c_{m,n_1,n_2}\phi_{m,n_1,n_2}(x,y) \\ & + \textstyle\sum_{n_1}\sum_{n_2} h_{m,n_1,n_2}\psi^h_{m,n_1,n_2}(x,y) \\ & + \textstyle\sum_{n_1}\sum_{n_2} v_{m,n_1,n_2}\psi^v_{m,n_1,n_2}(x,y) \\ & + \textstyle\sum_{n_1}\sum_{n_2} d_{m,n_1,n_2}\psi^d_{m,n_1,n_2}(x,y) \end{aligned} \quad (2)$$

where ψ^h, ψ^v and ψ^d correspond to three different two-dimensional wavelets with particular orientation, respectively: horizontal, vertical and diagonal. The c_{m,n_1,n_2} coefficients are the coarse resolution approximation, while h_{m,n_1,n_2}, v_{m,n_1,n_2} and d_{m,n_1,n_2} are the detail coefficients of the image approximations. By recursivity, the coefficients' values of the lower resolution levels are computed in a similar way and

$$\begin{aligned} WT = & \Big\{\{h_{m,n_1,n_2}\}_{n_1,n_2}, \{v_{m,n_1,n_2}\}_{n_1,n_2}, \\ & \{d_{m,n_1,n_2}\}_{n_1,n_2}, \{h_{m-1,n_1,n_2}\}_{n_1,n_2}, \\ & \{v_{m-1,n_1,n_2}\}_{n_1,n_2}, \{d_{m-1,n_1,n_2}\}_{n_1,n_2}, \\ & ,\ldots\ldots, \\ & \{h_{m-M,n_1,n_2}\}_{n_1,n_2}, \{v_{m-M,n_1,n_2}\}_{n_1,n_2}, \\ & \{d_{m-M,n_1,n_2}\}_{n_1,n_2}, \{c_{m-M,n_1,n_2}\}_{n_1,n_2}\Big\} \end{aligned} \quad (3)$$

is obtained as the two-dimensional discrete wavelet transform for $M+1$ levels. Next, the matrices of WT values are split into eight by eight blocks. Each of them is transformed by applying the two-dimensional discrete cosine transform (DCT) in such a way that

$$\begin{aligned} & \hat{h}_{m,\theta_1,\theta_2,k_1,k_2} = 0.25a(k_1)a(k_2)\cdot \\ & \textstyle\sum_{\eta_1,\eta_2} h_{m,n_1,n_2} \cos\frac{(2\eta_1+1)k_1\pi}{16} \cos\frac{(2\eta_2+1)k_2\pi}{16} \end{aligned} \quad (4)$$

where
$a(k) = \begin{cases} 1/\sqrt{2} & \text{if} \quad k=0 \\ 1 & \text{if} \quad k \neq 0 \end{cases}$; $k_1, k_2, \eta_1, \eta_2 = 0, 1, \ldots, 7$; $n_1 = 8\theta_1 + \eta_1$; $n_2 = 8\theta_2 + \eta_2$ and θ_1, θ_2 are block numbers. In a similar way the next values are computed and

finally we obtain the set

$$
\begin{aligned}
WCT = \Bigg\{ & \left\{ \left\{ \hat{h}_{m,\theta_1,\theta_2,k_1,k_2} \right\}_{k_1,k_2} \right\}_{\theta_1,\theta_2}, \\
& \left\{ \left\{ \hat{v}_{m,\theta_1,\theta_2,k_1,k_2} \right\}_{k_1,k_2} \right\}_{\theta_1,\theta_2}, \\
& \left\{ \left\{ \hat{d}_{m,\theta_1,\theta_2,k_1,k_2} \right\}_{k_1,k_2} \right\}_{\theta_1,\theta_2}, \\
& ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\
& \left\{ \left\{ \hat{h}_{m-M,\theta_1,\theta_2,k_1,k_2} \right\}_{k_1,k_2} \right\}_{\theta_1,\theta_2}, \\
& \left\{ \left\{ \hat{v}_{m-M,\theta_1,\theta_2,k_1,k_2} \right\}_{k_1,k_2} \right\}_{\theta_1,\theta_2}, \\
& \left\{ \left\{ \hat{d}_{m-M,\theta_1,\theta_2,k_1,k_2} \right\}_{k_1,k_2} \right\}_{\theta_1,\theta_2}, \\
& \left\{ \left\{ \hat{c}_{m-M,\theta_1,\theta_2,k_1,k_2} \right\}_{k_1,k_2} \right\}_{\theta_1,\theta_2} \Bigg\}
\end{aligned}
\qquad (5)
$$

which consists of values of the discrete wavelet-cosine transform (WCT) for image (1). Fig.1 presents the signal transmission system in which the WCT lossy compression was applied.

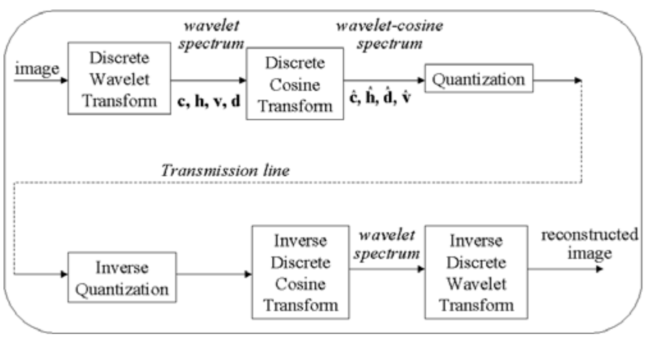

Figure 1: System of compressed image transmission

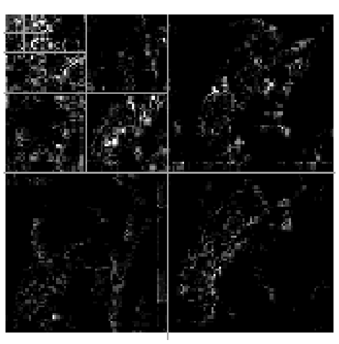

Figure 2: Wavelet transform of Lena image

3 LOSSY COMPRESSION

The WCT can be used in lossy compression systems in a way similar to the one in which the cosine transform is used in compression standards. The main idea of lossy compression lies in quantization with smaller accuracy. The original WCT uses 8 bits accuracy. Due to the fact that WCT belongs to the group of frequency transforms, the proper nonuniform quantization is easy to find.

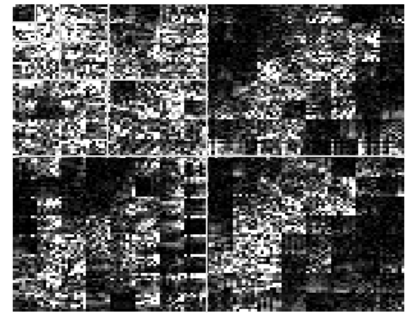

Figure 3: Wavelet-cosine transform of Lena image

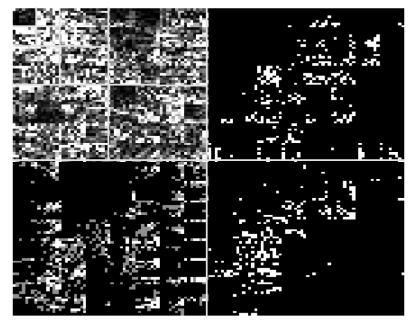

Figure 4: Compressed wavelet-cosine transform (nonuniform quantization)

An example of the WCT application to the lossy compression is presented in Fig.2 - Fig.9 for the Lena image. The Daubechies' wavelet of 12-th order was used as the kernel of the discrete wavelet transform. The wavelet transform for four resolution levels is presented in Fig.2. The wavelet-cosine transform for this case is presented in Fig.3, where the 8 bits uniform quantization was applied. In Fig.4, the compressed WCT spectrum with nonuniform quantization is presented. The low frequency level c_{m-3} has 8 bits accuracy, $h_{m-3}, v_{m-3}, d_{m-3}$ have 7 bits, $h_{m-2}, v_{m-2}, d_{m-2}$ and $h_{m-1}, v_{m-1}, d_{m-1}$ have 6 bits and h_m, v_m, d_m have only 2 bits. In this way, we try to stress the low frequencies of the image and to attenuate the high frequencies. Spectrum values for the high frequencies are represented with small bit accuracy but the number of samples is greater for the high than for the low frequencies. These two opposing phenomena make the bit stream for the high frequencies larger than for the low frequencies. The average number of bits per WCT value is given by formula

$$\underline{b} = 2^{-2(M+1)} b(\hat{c}_{m-M}) + \sum_{i=m-M}^{m} 2^{2(i-m-1)} \left[b(\hat{h}_i) + b(\hat{v}_i) + b(\hat{d}_i) \right] \tag{6}$$

where $b(\hat{c}_{m-M})$, $b(\hat{h}_i)$, $b(\hat{v}_i)$ and $b(\hat{d}_i)$ are numbers of bits per WCT value for decomposition level $\hat{c}_{m-M}$ and levels $\hat{h}_i$, $\hat{v}_i$ and $\hat{d}_i$ respectively, where $i = m-M, ..., m$. For the presented in Fig.4 case $M = 3$, we obtain

$$\underline{b} = 2^{-8}8 + 2^{-8}3 \cdot 7 + 2^{-6}3 \cdot 6 + 2^{-4}3 \cdot 6 + 2^{-2}3 \cdot 2 \approx 3.0 \tag{7}$$

bits/(WCT value). The bit stream consists of five parts and each of them has the following fraction (from the lowest to the highest frequency): 1.03% + 2.72% + 9.31% + 37.26% + 49.68%. It means that despite of the low accuracy, only 2 bits/(WCT value), the bit stream connected with the highest frequency takes almost 50% of the stream capacity.

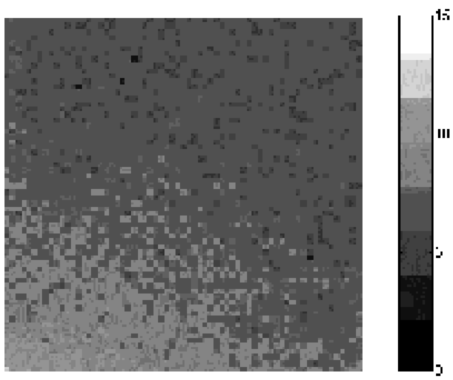

Figure 5: Fourier spectrum of the original image

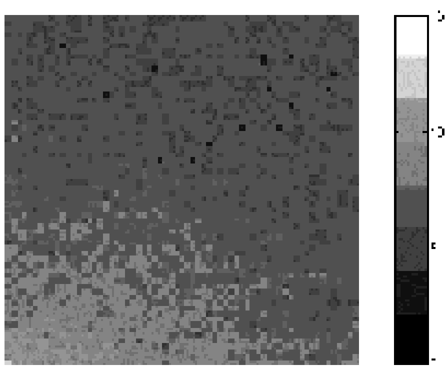

Figure 6: Fourier spectrum of the compressed image (uniform quantization)

To recover the image from the compressed WCT we proceed in the opposite direction. The inverse discrete cosine transform is used first and afterwards the inverse wavelet transform. In this way the compressed image is

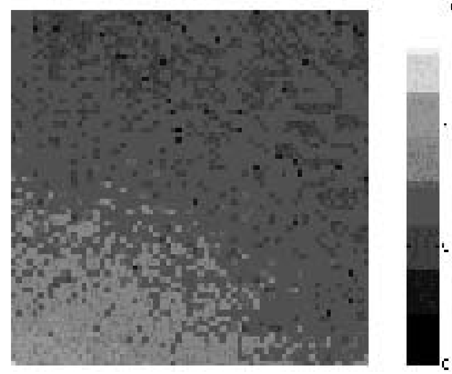

Figure 7: Fourier spectrum of the compressed image (nonuniform quantization)

obtained. If we compare the spectrum of original image presented in Fig.5 with the spectra (see Fig.6 and Fig.7) of the compressed images, quantization distortions proportional to the sampling frequency are not visible. The experiments carried out showed that such kind of noise is not observed for the 2-D signals while it is important for the 1-D signal case as reported in [5]. Fig.8 and Fig.9 present the Fourier spectra for images which were obtained as differences between the original and the compressed images with uniform quantization (Fig.8) and nonuniform quantization (Fig.9). In both cases the greatest distortions appeared for the lowest frequencies. It suggests that the sampling rate, 8 bits per WCT spectrum value, is too small and causes deformations in the reconstructed image. Uniform quantization leads to distortions which are spread proportionally to the inverse frequency (see Fig.8). For the case with nonuniform quantization (see Fig.9) the distortions for the low frequencies are apparently smaller then for the high frequencies.

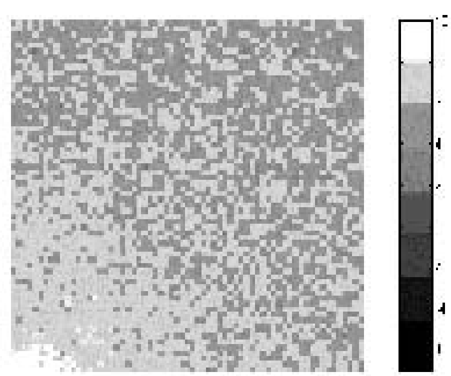

Figure 8: Fourier spectrum of the difference between the original and the compressed image (uniform quantization)

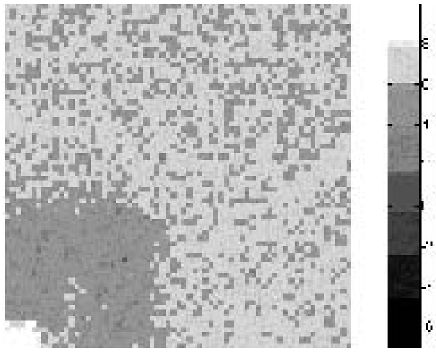

Figure 9: Fourier spectrum of the difference between the original and the compressed image (nonuniform quantization)

4 CONCLUSIONS

Some test images (see Table 1) were used to verify the usefulness of WCT in the image compression. Its efficiency was compared with the efficiency of wavelet transform. In all cases the average 3 bits accuracy was kept. Signal to noise ratio was used as an objective measure to evaluate the performance of the compression method. The wavelet-cosine transform led to the slightly smaller image distortions than the wavelet transform did. The usefulness of both transforms distinctly increased for the reasonable nonuniform quantization.

The WCT gives the wide variety of the nonuniform quantizations. The quantization in WCT domain was presented in this paper, however the more interesting results can be obtained if the nonuniform quantization is carry out in two steps. Firstly, just after the wavelet transformation and next, after the cosine transformation. Such a lossy compression method which consists of two steps, enables us to find a more effective method which allow us to take into account more properties of the compressed signals.

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Table 1: Signal to noise ratio for different compression method [dB]

Images	WT	
	uniform quantization	nonuniform quantization
Lena	20.3	48.7
woman	20.9	45.3
geometry	5.2	14.6
chess	22.3	50.4
binary bar	0.7	20.2
	WCT	
	uniform quantization	nonuniform quantization
Lena	20.2	49.3
woman	20.8	46.4
geometry	6.3	14.8
chess	22.3	51.8
binary bar	14.8	35.7

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